

Investigating the Role of Real Divisia Money in Persistence-Robust Econometric Models

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Abstract

This paper investigates the causal relationships between real money and real activity. Whereas previous literature has mainly focused on simple-sum aggregates, we instead use Divisia ones, thus avoiding the so-called Barnett Critique. Standard Granger non-causality tests are implemented in two different frameworks: Fully Modified VAR's (Phillips, 1995) and surplus-lag VARX models (Bauer and Maynard, 2012). These two environments allow modeling mixtures of $I(0)/I(1)$ variables with possible cointegration without pretesting for integration nor for the dimension of the cointegration space. Moreover the latter method is also robust to various other forms of persistence such as local-to-unity processes, long memory/fractional integration, or unmodeled breaks-in-mean in the causal variables. By implementing the tests on different sub-samples identified by standard structural break tests, and using three different measures of money (DM4, DM4- and DM3), the tests suggest a unidirectional causality from activity to money. Moreover, from one period to another, the whole causal structure of the systems seem to change, as well as the stationarity of the series. At last, the two methodologies return similar results.

1 Introduction

Economists have long been interested in the role of money in the economy, especially regarding its role as a leading indicator for future activity. Following the early works of Sims (1972, 1980), most researchers have focused on non-causality tests *à la* Granger (1969) in Vector Auto-Regressions (VAR). Among many others, Hayo (1999) and Walsh (2003) survey this vast literature, both emphasizing the contradictory results found by many researchers, showing that it is not possible to draw clear conclusions about the role of money in the economy. A similar conclusion was previously reached by Stock and Watson (1989), noticing that “Researchers using only slightly different specifications have reached disconcertingly different conclusions”.

In this paper, we argue that such instability of results may have at least two distinct sources: i) Measurement errors in monetary aggregates due to inappropriate aggregation methods and, ii) Two-stage inference in econometric models.

Concerning the former point, as suggested in a number of seminal publications (Barnett, 1980; Barnett, Offenbacher and Spindt, 1981, 1984; Barnett and Serletis, 2000), there exists an internal inconsistency between the microeconomics used to model the private sector, and the aggregator functions used to compute monetary aggregates by central banks. As a corollary, simple-sum aggregates, computed by central banks, are likely to return flawed measures of money, being based on the unrealistic assumption of perfect substitutability of assets. This critique, known as the Barnett Critique, defined by Chrystal and MacDonald (1994), strongly affects the results given by macroeconomic models, implying that outcomes of studies using simple-sum aggregates are misleading. Belongia (1996) has investigated the empirical validity of this Critique by reexamining several studies on money, replacing the flawed simple-sum measures with theoretically-consistent ones. He shows that the original conclusions are then deeply altered. Theoretically-consistent measures of money have been used in a number of studies such as Barnett, Fisher and Serletis (1992), Chrystal and MacDonald (1994) who study the relationship between money and GDP, Schunk (2001) focusing on the predictive content of money, Serletis (1991) and Serletis and King (1993) or Yue and Fluri (1991). Other recent works also include Darrata et al. (2005), Barnett, Chauvet and Tierney (2009), Belongia and Ireland (2012a, 2012b) or Rahman and Serletis (2012).

Concerning the latter point, classical tests of non-causality rely on pre-tests (first-stage inference) on the persistence of the series. For instance, if all series are stationary, a simple VAR in levels is estimated, whereas if the series are integrated but not cointegrated, a VAR in first

differences is used. If additionally, cointegration is present, then a Vector Error Correcting Model (VECM) is estimated (Toda and Phillips 1993, 1994). This simple example emphasizes a general method consisting of: i) Trying to identify the correct underlying Data Generating Process (DGP) according to pre-tests for stationarity/cointegration (first-stage inference), and ii) Testing for non-causality in the estimated DGP (second-stage inference). Nevertheless, in empirical work, due to lack of power of stationarity tests, estimating the “true” DGP may be difficult, as well as differentiating between the different kind of persistence as local-to-unity processes, (Phillips, 1987; Chan, 1988), or long memory/fractional integration; a problem that deeply worsens if structural breaks are present (Diebold and Inoue, 2001). Hence, inappropriate persistence modelling, and thus inaccurate DGP identification may lead to incorrect inference and cascading errors in non-causality tests.

The goal of the paper is to study the causal relationships between real money and activity by taking into account the two above points. To avoid the Barnett Critique, we use theoretically-consistent measures of money, i.e. Divisia indices. Concerning the econometric models, we follow Bauer and Maynard (2012), suggesting agnostic approaches to the form of persistence, that is suggesting tests that are robust to the various forms of persistence, without any pre-testing. In causality testing, to our knowledge, two approaches can be used, i) The Fully-Modified VAR (FM-VAR) of Phillips (1995), extending the Fully Modified ordinary least square of Phillips and Hansen (1990), ii) The surplus-lag VARX model of Bauer and Maynard (2012), based on Toda and Yamamoto (1995), Dolado and Lütkepohl (1996) or Saikkonen and Lütkepohl (1996). Whereas the FM-VAR is designed to model a mixture of $I(0)/I(1)$ variables with possible but undefined cointegration, the Bauer and Maynard (2012) surplus-lag VARX approach possesses basically the same features, but goes further. Indeed, the causality test remains robust, when the causal variable has long memory/fractional integration, can be modeled as a local-to-unity process, or contains a certain amount of unmodeled breaks in mean. In this latter case, the noncausality test is interpreted as a co-breaking test.

The main results of the paper are twofold: i) Using three different Divisia monetary aggregates, tests support an unilateral causality from real activity to real money, ii) Over the different sub-periods, the whole causal structure of the systems, as well as the long term relationships seem to change.

This paper is organized as follows. Section two focuses on monetary aggregation and presents the theoretically consistent Divisia aggregates. Section three introduces the econometric methodologies. In Section 4, we first test for structural breaks on the growth rates of the series, and

then implement non-causality tests on different sub-periods. At last Section 5 concludes and discusses the results.

2 The theoretical approach to monetary aggregation

In this section, we briefly recall the theoretical approach to monetary aggregation. Let $x_t = (x_{1t}, \dots, x_{kt})'$ and $m_t = (m_{1t}, \dots, m_{pt})'$ be respectively two vectors of real consumption goods and real monetary assets in period $t, t = 1, \dots, T$. Let l_t be leisure. Assume that data are rationalized by a well-behaved utility function:

$$U_t = U(x_t, m_t, l_t) \quad (1)$$

i.e. each bundle $(x_{1t}, \dots, x_{kt}, m_{1t}, \dots, m_{pt}, l_t)'$ is the solution of the utility maximization program¹, where the nominal price of money, for an asset m_{it} is defined according to Barnett (1978):

$$p_{it}^n = p_t \frac{R_t - r_{it}}{1 + R_t}, i = 1, \dots, k \quad (2)$$

Where:

R_t is a benchmark rate,

r_{it} is the asset's own rate,

p_t is a consumer price index.

To allow for aggregation, further assume that the utility function (1) is weakly separable over the monetary assets, and thus admits a rewriting:

$$U_t = U(x_t, m_t, l_t) = V(x_t, U_1(m_t), l_t) \quad (3)$$

Where:

$V(\cdot)$ is a strictly increasing function, known as the macro-function,

$U_1(\cdot)$ is the monetary sub-utility function, known as the micro-function, which, if homothetic, is also the aggregator function.

The above weakly separable utility structure ensures that an aggregate exists over money². Following Diewert (1976, 1978), if the preferences over the monetary assets are homothetic, then

¹For theoretical justifications of money in the utility function, see Feenstra (1986) and Poterba and Rotemberg (1987).

²Notice that here weak separability is assumed but not tested. For more formal testing procedures, see for example Varian (1983), Swofford and Withney (1987, 1988), de Peretti (2007), Barnett and de Peretti (2009), or Fleissig and Whitney (2003, 2005).

$U_1(\cdot)$ is interpreted as the aggregator function. An index number $Q(m_{t+1}, m_t, p_{t+1}^n, p_t^n)$, must therefore satisfy:

$$Q(m_{t+1}, m_t, p_{t+1}^n, p_t^n) = \frac{U_1(m_{t+1})}{U_1(m_t)} \quad (4)$$

If preferences are non-homothetic, the aggregator function is no longer the sub-utility function, but the distance function. The distance function $D(\bar{U}_1, m_t) = \max_l \{l : U(m_t/l) \geq \bar{U}_1\}$ returns by what proportion one has to inflate or deflate the vector m_t in order to reach the utility level \bar{U}_1 . A consistent aggregate is given by:

$$Q(m_{t+1}, m_t, p_{t+1}^n, p_t^n) = \frac{D(\bar{U}_1, m_{t+1})}{D(\bar{U}_1, m_t)} \quad (5)$$

Diewert (1976, 1978, 1980) proved that in both case cases $Q(m_{t+1}, m_t, p_{t+1}^n, p_t^n)$ can be nonparametrically and consistently estimated as:

$$Q(m_{t+1}, m_t, p_{t+1}^n, p_t^n) = \prod_{i=1}^p \left[\frac{m_{it+1}}{m_{it}} \right]^{(s_{it+1} + s_{it})/2} \quad (6)$$

Where:

s_{it} is the budget share for the monetary asset i in period t ,

$$\bar{U}_1 = \sqrt{U_1(m_{t+1})U_1(m_t)}.$$

Clearly, (6) is a discrete approximation of the continuous Divisia index.

In their seminal paper, Barnett, Offenbacher and Spindt (1984) present a very comprehensive survey of the so-called Divisia aggregation. They show that very different conclusions are drawn when one considers Divisia aggregation, rather than simple-sum aggregates, the latter being valid only if the assets are substitutes, which is an unrealistic assumption. As an illustration, Figure (1) plots two monetary aggregates: The DM4 (see below), and the simple-sum built by adding the component levels of the DM4 assets. Both aggregates are divided by a consumer price index and are in log-form. On the same figure, we plot (right scale) the aggregation bias, which is computed as the difference between the two indices. Clearly, the error is not constant over time and dramatically increases up to the early 90's, according to the explosion of financial innovations. After the early 90's, the bias decreases, and then constantly increases at a lower growth rate. Notice that the bias reflects different trends in the two series. The time-varying nature of the bias implies that simple-sum aggregates can not be used as proxies of Divisia aggregates.*

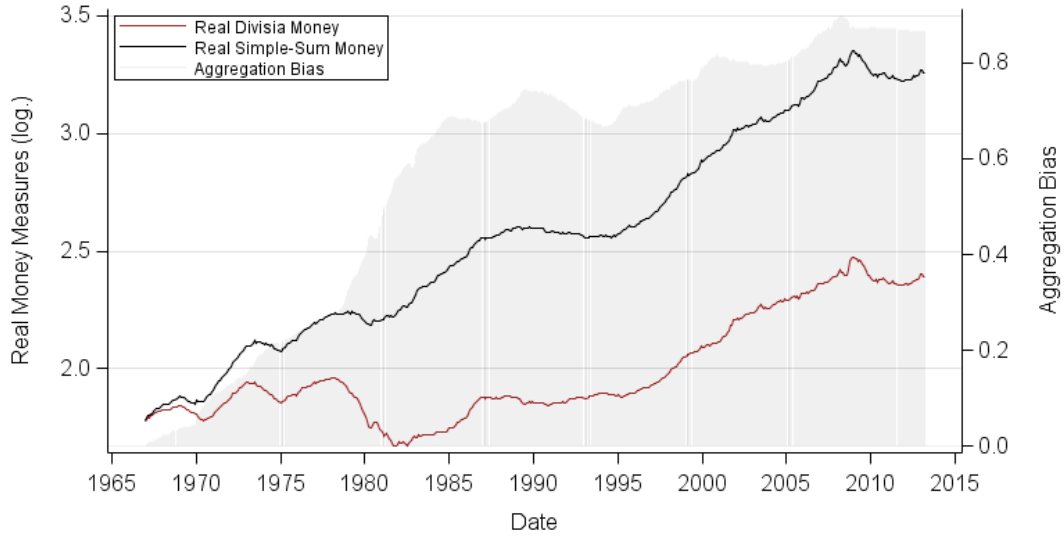


Figure 1: Real DM4, simple-sum over the same components as the DM4, aggregation bias. (shaded area, right scale).

3 Non-causality tests in persistence-robust econometric models

To analyze the role of Divisia money in the economy, we use standard Granger non-causality tests in VAR models. Two frameworks are considered that do not impose pre-testing for persistence nor for cointegration: The FM-VAR approach of Phillips (1995), and the surplus-lag VARX model of Bauer and Maynard (2012). The former approach allows modeling a mixture of $I(0)/I(1)$ variables without specifying the order of integration of each component, nor the dimension of the cointegration space. The latter framework is more general since it also takes into account other forms of persistence in the causal variable as local-to-unity processes, fractional integration/ long memory and unmodeled structural breaks-in-mean in the causal variable. We first describe the FM-VAR method.

3.1 The Fully-Modified approach to VAR modeling

Let y_t be a k -vector time series generated by the finite-order VAR of order p , $\text{VAR}(p)$:

$$y_t = J(L)y_{t-1} + \varepsilon_t, t = 1, 2, \dots, T \quad (7)$$

Table 1: $SupF_T(l+1|l)$ sequential tests for structural breaks, monetary components

| $\Delta \text{div}4r_t, \text{Lag} = 2, h = 0.1$ | | | |
|--|-----|-----------------|------------------|
| $l+1$ | l | $SupF_T(l+1 l)$ | $Cut - off(5\%)$ |
| 1 | 0 | 23.22 | 14.60 |
| 2 | 1 | 14.37 | 16.53 |
| 3 | 2 | 08.57 | 17.43 |

Final (global) Estimates of the Single Break:
Sep1981

| $\Delta \text{div}4mr_t, \text{Lag} = 2, h = 0.1$ | | | |
|---|-----|-----------------|------------------|
| $l+1$ | l | $SupF_T(l+1 l)$ | $Cut - off(5\%)$ |
| 1 | 0 | 31.11 | 14.60 |
| 2 | 1 | 22.98 | 16.53 |
| 3 | 2 | 16.46 | 17.43 |

Final (global) Estimates of the Two Breaks:
Feb1996, Mar2008

| $\Delta \text{div}3r_t, \text{Lag} = 2, h = 0.1$ | | | |
|--|-----|-----------------|------------------|
| $l+1$ | l | $SupF_T(l+1 l)$ | $Cut - off(5\%)$ |
| 1 | 0 | 44.99 | 14.60 |
| 2 | 1 | 19.90 | 16.53 |
| 3 | 2 | 15.81 | 17.43 |

Final (global) Estimates of the Two Breaks:
Apr1995, Mar2008

where:

$$J(L) = \sum_{i=1}^p J_i L^{i-1},$$

$$\varepsilon_t \sim \text{Niid}(0, \Sigma_{\varepsilon\varepsilon}).$$

(7) can be equivalently re-written as:

$$y_t = B(L)\Delta y_{t-1} + Ay_{t-1} + \varepsilon_t \quad (8)$$

where:

$$B(L) = \sum_{i=1}^{p-1} B_i L^{i-1} \text{ with } B_i = - \sum_{j=i+1}^p J_j,$$

$$\text{and } A = J(1).$$

or as:

$$\Delta y_t = B(L)\Delta y_{t-1} + \Pi y_{t-1} + \varepsilon_t \quad (9)$$

where:

$$\Pi = A - I_k$$

This latter representation is the Vector Error Correction form representation of a VAR process. The A matrix or (II) contains all relevant information about the long-term relationships. Now, re-write model (8) as:

$$y_t = Bz_t + Ay_{t-1} + \varepsilon_t \quad (10)$$

$$= Fx_t + \varepsilon_t \quad (11)$$

Where:

$$z_t = (\Delta y'_{t-1}, \dots, \Delta y'_{t-p+1})'$$

$$x_t = (\Delta y'_{t-1}, \dots, \Delta y'_{t-p+1}, y'_{t-1})'$$

$$B = [B_1, B_2, \dots, B_{p-1}].$$

Or in matrix form as:

$$Y' = BZ' + AY'_{-1} + E' \quad (12)$$

$$= FX' + E' \quad (13)$$

Where:

$$F = [B_1, \dots, B_{p-1}, A].$$

Since the above model can not be efficiently estimated by Ordinary Least Square (OLS) due to a second order (endogeneity) bias, Phillips (1995) suggest applying the Fully Modified estimator (Phillips and Hansen, 1990) to the system. Define $\hat{\Omega}_{\varepsilon y}$ and $\hat{\Omega}_{yy}$ as estimates of the two-sided long-run covariance matrices of respectively $\eta_t = (\hat{\varepsilon}_t = y_t - Fx_t, \Delta y_{t-1})$ and Δy_{t-1} , where $\hat{\varepsilon}_t$ are the OLS residuals of model (8). Also define $\hat{\Delta}_{\varepsilon \Delta y}$ and $\hat{\Delta}_{\Delta y \Delta y}$ as estimates of the one-sided long-run covariances matrices of respectively $\eta_t = (\hat{\varepsilon}_t = y_t - Fx_t, \Delta y_{t-1})$ and Δy_{t-1} . Notice that both types of matrices are kernel estimates. For instance $\hat{\Omega}_{\varepsilon y}$ and $\hat{\Delta}_{\varepsilon \Delta y}$ are computed as:

$$\hat{\Omega}_{\varepsilon y} = \sum_{j=-T+1}^{T-1} w(j/k1) \hat{\Gamma}(j) \quad (14)$$

$$\hat{\Delta}_{\varepsilon \Delta y} = \sum_{j=0}^{T-1} w(j/k1) \hat{\Gamma}(j) \quad (15)$$

Where:

$w(j/k1)$ is a kernel smoothing function,

$k1$ is the bandwidth or truncation parameter,

$\hat{\Gamma}(j)$ is the standard covariance estimator, $T^{-1} \sum_{t=1}^T \eta_t \eta'_{t+j}$

The FM-VAR estimator is then defined as:

$$\hat{F}^+ = [Y'Z | Y'Y_{-1} - \hat{\Omega}_{\varepsilon y} \hat{\Omega}_{yy}^{-1} (\Delta Y'_{-1} Y_{-1} - T \hat{\Delta}_{\Delta y \Delta y})] (X'X)^{-1} \quad (16)$$

Where:

\parallel is the horizontal stacking operator,

$$\Delta Y'_{-1} = Y'_{-1} - Y'_{-2}.$$

Compared to the standard FM estimator for OLS, no correction for autocorrelation is made.

Thus, the only correction is for endogeneity.

Defining $\hat{F} = [Y'Z][Y'Y_{-1}](X'X)^{-1}$ as the simple OLS estimator, (16) can be re-written in order to make apparent the correction for endogeneity:

$$\hat{F}^+ = \hat{F} - [\hat{\Omega}_{\varepsilon y} \hat{\Omega}_{yy}^{-1} (\Delta Y'_{-1} Y_{-1} - T \hat{\Delta}_{\Delta y \Delta y})] (X'X)^{-1} \quad (17)$$

A non-causality test from variable j in equation i , amounts to testing:

$$B_1[ij] = \dots = B_{p-1}[ij] = A[ij] = 0$$

which can be stated as:

$$H_0 : Rvec(\hat{F}^+) = 0 \quad (18)$$

where R is a $(q \times 1)$ suitable selection matrix, with here $q = p$. Then the corresponding Wald test is given by:

$$W^+ = T(Rvec(\hat{F}^+))' [R(\hat{\Sigma}_{\varepsilon} \otimes T(X'X)^{-1})R']^{-1} (Rvec(\hat{F}^+)) \quad (19)$$

Phillips (1995) proved that under the null, W^+ is asymptotically bounded by a Chi-squared distribution with q degrees of freedom, q being the rank of R .

3.2 The surplus-lag VARX model

Alternatively, one may also consider surplus-lag models to test for non-causality between two variables, y_{1t} the dependent one and y_{2t} the exogenously modeled forcing (causal) variable, given a set $z_t = (y_{3t}, \dots, y_{kt})'$ of control variables. We here follow Bauer and Maynard (2012), extending and simplifying Toda and Yamamoto (1995). In our framework, i.e. the causality between two series, write down and estimate the single equation of interest of a surplus-lag VARX(p, p_1), here a Auto-Regressive ARX(p, p_1) process:

$$y_{1t} = \sum_{j=1}^p (\psi_j^1 y_{1t-j} + \psi_j^2 z_{t-j}) + \sum_{j=1}^{p_1+1} \psi_j^3 y_{2t-j} + \varepsilon_t \quad (20)$$

and jointly test for $\psi_1^3 = \psi_2^3 = \dots = \psi_{p_1}^3 = 0$ using a standard Wald tests³.

³See Bauer and Maynard (2012) for computational details.

The two models are clearly complimentary. On the one hand, if series are $I(0)/I(1)$ and cointegration is present, the FM-VAR is efficient, unlike the VARX which forces an extra lag. Nevertheless, as shown by Bauer and Maynard (2012) based on Monte Carlo simulations, efficiency losses have a very limited impact on the power of causality tests. On the other hand, for local-to-unity processes, the causality tests in FM-VAR's may not behave well, as shown by Yamada and Toda (1997), which is a case for which the VARX is designed for. Hence, different informations between the two tests may indicate different kinds of persistence. For other kinds of persistence, such as long memory, nothing is known for the FM-VAR.

We next turn to implementations⁴.

4 Implementing the tests

The role of real money balance effects has recently been reconsidered by Favara and Giordani (2009), using a structural VAR, imposing restrictions consistent with the New Keynesian framework. They suggest that real money balances shocks may have a significant effect on output and prices. Dorich (2009) reaches a very similar conclusion using a money-in-the-utility model. Interestingly, he concludes that such effects may be based on a non-separable utility function, a fundamentally different framework from ours. Those conclusions are in sharp contrast with Woodford (2003) and Ireland (2004), who show the negligible impact of real money. All these articles used inconsistent measures of money. Here, we reconsider their conclusions using three different theoretically-consistent measures of broad money. We first describe the data and analyze their statistical properties in terms of structural breaks, and then implement the described non-causality tests.

4.1 Data

Let $y_t^1 = (div4r_t, indpro_t, irst_t, irlt_t, \Delta_{12}p_t)'$, $y_t^2 = (div4mr_t, indpro_t, irst_t, irlt_t, \Delta_{12}p_t)'$ and $y_t^3 = (div3r_t, indpro_t, irst_t, irlt_t, \Delta_{12}p_t)'$ be k -vectors of variables being integrated of order 1 or 0, where: $div4r_t$, $div4mr_t$ and $div3r_t$ are respectively the logarithms of the real Divisia DM4, DM4- and DM3 indices; $indpro_t$ is the logarithm of the Industrial Production Index, used as a proxy of the real activity; $irst_t$ and $irlt_t$ are respectively short (three-month) and long-term (one year) interest rates and $\Delta_{12}p_t$ is the Consumer Price Index inflation rate. All data are on a monthly basis, and span, for the United States, the period covering January 1968 to December 2012. Data

⁴All the routines, i.e the FM-VAR model, the surplus-lag VARX as well as the Bai and Perron (1998) tests (see below) are programmed using SAS/IML and are available under request at the corresponding author email adress: philippe.de-peretti@univ-paris1.fr

Table 2: $SupF_T(l+1|l)$ sequential tests for structural breaks, Other components

| $\Delta(\Delta_{12}p_t), Lag = 3, h = 0.1$ | | | |
|--|-----|-----------------|----------------|
| $l+1$ | l | $SupF_T(l+1 l)$ | $Cut-off(5\%)$ |
| 1 | 0 | 43.40 | 16.76 |
| 2 | 1 | 21.92 | 18.56 |
| 3 | 2 | 19.40 | 19.53 |
| 4 | 3 | 10.75 | 20.24 |

Final (global) Estimates of the Three Breaks:
Nov1982, Oct1991, Oct2007

| $\Delta indpro_t, Lag = 3, h = 0.1$ | | | |
|-------------------------------------|-----|-----------------|----------------|
| $l+1$ | l | $SupF_T(l+1 l)$ | $Cut-off(5\%)$ |
| 1 | 0 | 34.96 | 16.76 |
| 2 | 1 | 09.42 | 18.56 |
| 3 | 2 | 07.96 | 19.53 |

Final (Global) Estimates of the Single Break:
Dec1981

| $\Delta irst_t, Lag = 1, h = 0.1$ | | | |
|-----------------------------------|-----|-----------------|----------------|
| $l+1$ | l | $SupF_T(l+1 l)$ | $Cut-off(5\%)$ |
| 1 | 0 | 07.39 | 12.25 |
| 2 | 1 | 04.25 | 13.83 |
| 3 | 2 | 04.01 | 14.73 |

| $\Delta irlt_t, Lag = 2, h = 0.1$ | | | |
|-----------------------------------|-----|-----------------|----------------|
| $l+1$ | l | $SupF_T(l+1 l)$ | $Cut-off(5\%)$ |
| 1 | 0 | 12.09 | 14.06 |
| 2 | 1 | 08.38 | 13.83 |
| 3 | 2 | 06.50 | 14.73 |

are collected from the Federal Reserve of St Louis⁵, except the Divisia indices computed by the Center for Financial Stability⁶ (CFS). The CFS monthly reports statistics for Divisia indices as the broad DM4, DM4-, which is DM4 without short term treasury bills, and DM3, which does not include treasuries nor commercial paper, corresponding to the discontinued M3 aggregate. Barnett et al. (2012) describe the construction of those monetary aggregates. The nominal price of money for each asset is computed using a corresponding interest rate and a benchmark rate; interest bearing checking accounts are paired with the national average interest rate on those accounts, for example. The benchmark rate is chosen as a maximum rate among the ‘basket’ of component rates along with other comparable loan rates. Included in this basket is a short

⁵<http://research.stlouisfed.org/fred2/>

⁶www.centerforfinancialstability.org

Table 3: $SupF_T(l+1|l)$ sequential tests for structural breaks in variances for the two interest rates

| $\Delta_{irst_t}, Lag = 0, h = 0.1$ | | | |
|-------------------------------------|-----|-----------------|----------------|
| $l+1$ | l | $SupF_T(l+1 l)$ | $Cut-off(5\%)$ |
| 1 | 0 | 78.12 | 12.25 |
| 2 | 1 | 13.25 | 13.83 |
| 3 | 2 | 12.66 | 14.73 |

Global Estimates of the Two Breaks:
Sep1982, Nov2007

| $\Delta_{irlt_t}, Lag = 1, h = 0.1$ | | | |
|-------------------------------------|-----|-----------------|----------------|
| $l+1$ | l | $SupF_T(l+1 l)$ | $Cut-off(5\%)$ |
| 1 | 0 | 22.13 | 12.25 |
| 2 | 1 | 13.35 | 13.83 |
| 3 | 2 | 12.71 | 14.73 |

Global Estimates of the Two Breaks:
Jul1979, Nov2007

Table 4: Sub-Periods for the non-causality tests

| Sub-Periods for y_t^1 | | |
|-------------------------|------------------------|--------------------|
| Sub-Period | Points in the Database | Dates |
| 1 | [1, 139[| [Jan1968, Jul1979[|
| 2 | [179, 286[| [Nov1982, Oct1991[|
| 3 | [286, 478[| [Oct1991, Oct2007[|
| 4 | [479, 539] | [Nov2007, Dec2012] |

| Sub-Periods for y_t^2 | | |
|-------------------------|------------------------|--------------------|
| Sub-Period | Points in the Database | Dates |
| 1 | [1, 139[| [Jan1968, Jul1979[|
| 2 | [179, 286[| [Nov1982, Oct1991[|
| 3 | [286, 338[| [Oct1991, Feb1996[|
| 4 | [338, 478[| [Feb1996, Oct2007[|
| 5 | [479, 539] | [Nov2007, Dec2012] |

| Sub-Periods for y_t^3 | | |
|-------------------------|------------------------|--------------------|
| Sub-Period | Points in the Database | Dates |
| 1 | [1, 139[| [Jan1968, Jul1979[|
| 2 | [179, 286[| [Nov1982, Oct1991[|
| 3 | [328, 478[| [Apr1995, Oct2007[|
| 4 | [479, 539] | [Nov2007, Dec2012] |

Note: Samples with less than 50 observations are not considered

term lending rate to commercial and industrial firms, a suggested in Offenbacher and Shachar (2011), who use the loan rate as the maximum hypothetical interest sacrificed for the liquidity service of the real assets. The loan rate is commonly the highest and therefore the benchmark rate compared. Depending on the period there are between fourteen and seventeen component asset and interest rate pairs included in the broadest possible aggregate, Divisia M4 (DM4) to account for entry and exit of assets, changes in survey methodology, and innovation in the financial markets. Money market demand accounts (MMDA) do not enter into the survey until the early 1980s, and then exit the survey in 1991 as they are folded into the more general savings accounts survey that exists throughout the entire period. Given the above discrete approximation methodology a consistent Divisia aggregate is then produced for econometric analysis.

Over such a long period, one can expect structural breaks to occur in either the mean or the variance (or in both) of the series, especially in their growth rates. Such breaks have been reported by a number of authors as Guégan and de Peretti (2013). Moreover, testing for structural breaks in first differences is of particular importance when dealing with FM-VAR models. Indeed, recall that such a methodology requires computing long-run covariances matrices, using both residuals and first differences of series.

To test for structural breaks, we proceed in two steps. We first use the procedure suggested in Bai and Perron (1998) to find the optimal number of breaks, by sequentially computing the $SupF_T(l+1|l)$ statistics, where l is the number of breaks under the null. For a given series, we begin by computing $SupF_T(1|0)$, to test for no break against a single break. If the null is rejected, we then add an additional break for one of the two segments, compute $SupF_T(2|1)$, and so on, until we fail to reject the null of no additional break. Then, taking l as the number of breaks, in a second step, we compute the breaking dates using a global minimizer. Tables (1, 2) report the results for a pure structural change model for a window $h = 0.1$. At the 5% threshold, all series, except the two interest rates, exhibit one or several breaks. The broadest monetary aggregate exhibit a clear single break in September 1981, coinciding with a contraction in broad money growth. The sub-aggregates DM3 and DM4- differ from the broader DM4 in that they experience a break in early 2008. This specific break does not coincide with any major revision in the monetary data collection methodology of the Federal Reserve⁷, but does coincide with the rate cuts enacted by the Federal Reserve in September and October of 2007 as well as a

⁷The closest major revision to the data methodology as described in Barnett et al (2013) is the discontinuation of M3 components including overnight repurchase agreements, requiring a new data source for such components. The discontinuation occurs in March of 2006, a little more than a year before the identified break in October 2007.

Federal Reserve program to conduct a large (\$100 billion) series of repurchase agreements over the course of twenty eight days in March of 2008. The monetary sub aggregates show an added break in April 1995 and February 1996, close to the beginning of the inclusion of retail sweeps in calculation of the Divisia⁸. A retail sweep occurs when banks reclassify checking deposits as savings deposits to skirt legal reserve requirements. Checking accounts are then underreported while savings accounts are over reported by a significantly large amount. Given the differing weights associated with each monetary component, the entrance of sweeps into the calculation and its subsequent take-off after 1996⁹, it is interesting that the break occurs during the period sweeps become relatively large (1995-1996) in the sub aggregates, but not the broadest: DM4.

For the inflation rate two breaks are found at the 5 % level and 3 at the 10% one. We thus adopt a 3-break model for the inflation rate model, which is line with Benati and Kapetanios (2003) using a slightly different methodology. For the growth rates of real activity, tests support a one-break model. Interestingly, activity and broad money exhibit a break close to the same time in late 1981.

For the two interest rates, no breaks in growth rates are observed. For these two series, we also tests for breaks in variance. As a rough analysis, we implement structural break tests on the square residuals of the two models defined in Table (2). Results are reported in Table (3). At the 5% threshold, both series exhibit a clear break, and at the 10% two breaks. In both cases, we conclude here in favor of a two-break model for the two series. Notice that both series exhibit a break in the late 2007, at nearly the same date.

Grouping the results of structural break tests suggests testing for non-causality on different sub-periods, presented in Table (4).

4.2 Non-causality in FM-VAR models

To estimate the various FM-VAR models¹⁰, we have to jointly choose two parameters: i) The lag p of the VAR model, ii) The bandwidth parameter $k1$, knowing that for $k1$ several intervals are given by Phillips (1995) that rule out automatic bandwidth selection procedures (see Andrews, 1991). To tackle this problem, we adopt a two-step data-driven procedure. First, using a double loop over p and $k1$, we estimate various FM-VAR models (8) on $p = 2, \dots, 10$, and for $k1 \in]1/4, 2/3[$ (see assumption BW p. 12 and theorem p 37), and keep the models having

⁸Unfortunately the retail sweep program for the St. Louis Fed has been discontinued. The data has not been available since March 2012, and those wishing to incorporate sweeps are left to estimate them as needed.

⁹See Anderson and Rasche (2001) for a more detailed description of sweeps programs.

¹⁰Following Andrews and Monahan (1992), a pre-whitened method is used in all models.

spherical disturbances. On a second run, among all these models, we select the one with the minimal *AIC* criterion (Akaike, 1974), and implement non-causality tests in this model. We report two kinds information: i) The estimates of the various long-term matrices A in Tables 5, 8 and 11, where the d variable signals if deterministic terms are added, where $d = 1$ means that an intercept is added, and $d = 2$ that an intercept plus a linear trend are included, and ii) The whole causal structure of the model. This allows analyzing if the long-term relations appear stable over time, as well as the stationarity of the variables, even if not formally tested. Specifications of the models are also presented. Tables (6), (9) and (12) present the results of non-causality tests for the different definitions of money.

Non-causality tests for DM4 Looking first at Table (5), it appears that the long term relationships change from a period to another, as the stationarity of the series. For instance, the long-term interest rate appears to be non-stationary in sub-period 1 but stationary in sub-periods 2 and 3. Picking an other example, Divisia money is also integrated of order one in sub-periods 1, 2 and 3. The sub-periods 4 being hard to interpret due to the non-nullity of the cross-coefficients. Turning to non-causality tests, Table (6), we fail to reject the null of non-causality from money to activity in all sub-periods, whereas activity clearly granger-causes money in the last three periods. For sub-period 1, causality is found only at the 10 % threshold. Concerning the other causal relationships, unexpectedly, the short-term interest rate seems to play no driving role concerning activity, but appears to be a causal variable for money. At last, note that, except for few exceptions, the whole causal structure of the systems change from one period to another.

Non-causality tests for DM4- As previously noticed, here again, the first striking point is that the long-term matrices are period-dependent, Table (8). Turning to non-causality tests, Table (9), in all sub-periods, the null of non-causality from money to real activity is not rejected. In the same time, real activity Granger-causes money, but the results depend on the time-period considered. The null is accepted for sub-periods 1 and 3, and deeply rejected over periods 2, 4 and 5. Over these two latter periods, the short-term interest rate is the driving force of real activity. The relation between the interest rate and money is also not constant over time, and we reach a very similar conclusion as for DM4: the causal structures of the systems change over time.

Non-causality tests for DM3 Focusing now on DM3, and only on causality tests as presented in Table (12), we have the same patterns: On sub-period 1, we have no causal relationships between money and activity. On sub-periods 2, 3 and 4, we reject non-causality from activity to money, while we fail to reject the null from money to activity.

Hence, whatever the definition of money used, the causal relationship for all models is clearly from activity to money, and not the converse.

4.3 Non-causality in surplus-lag VARX models

We next turn to results of non-causality tests in surplus-lag VARX models. Similarly to what we did for the FM-VAR lag selection, we use a double loop over p and $p1$, for $p, p1 = 1, \dots, 10$, estimate the various $ARX(p, p1)$ models, and keep those with spherical disturbances. In a second trial, among these models, we choose the one with the minimal AIC. Results are given by Tables (7), (10) and (13). We report the kind of model used, as well as the Wald statistic and the associated p-value, only for money and real activity.

Non-causality tests for DM4 Results using the broad monetary aggregate DM4, are similar with those obtained using the FM-VAR methodology. In sub-periods 2, 3 and 4 real activity is a causal variable of money. Results are less clear for sub-period 1 where causality is found only at the 10 % threshold. Interestingly, only in sub-period 2, the causal relation is bilateral, a result which is also suggested in Table (6).

Non-causality tests for DM4- Results are presented in Table (10). Concerning the null of non-causality from money to activity, it is accepted in all sub-periods. Focusing now on the causal relationship from activity to money it is clearly period-dependent, as within the FM-VAR framework: The null is rejected for sub-periods 2 and 5 (here at 10%), but accepted for other samples. Clearly, compared with DM4, the relationship is much more unstable.

Non-causality tests for DM3 At last, focusing on non-causality tests for DM3 returns exactly the same information as for tests implemented in FM-VAR models. For sub-period 1, no causal relationships are found between money and activity, whereas over other periods, the causal relationship is from activity to money. Hence, DM3 seems to behave as DM4.

5 Conclusion and discussion

In this paper, we have used Granger non-causality tests to investigate the empirical relationships between real money and activity. Three different broad measures of money as defined by the CFS have been used, DM4, DM4- and DM3. Tests have been implemented within two distinct frameworks: FM-VAR and surplus-lag VARX models. Results return important features: i) For all three aggregates, and for different sub-periods, tests suggest that if causality is found, it is from activity to real money, ii) In all models, there is no causal links between money and activity during the period January 1968 to July 1979, iii) For DM4-, the results are period-dependent, and the two aggregates showing a stable relationship with activity are the DM4 and DM3 ones, iv) From one sub-period to another, the whole causal structure of the systems, as well as the stationarity of some series seems to change, maybe also explaining the instability found previously by researchers, v) At last, the two methodologies return very similar information.

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6 Tables to be included in the paper

Table 5: Long-term matrices A for the four sub-periods. Money defined as $div4r_t$

| $y_t^1 = (div4r_t, indpro_t, irst_t, irlt_t, \Delta_{12}p_t)'$ | | | | | | | | | |
|---|--|--|--|--|---|--|--|--|--|
| Sub-period 1 | | | | | Sub-period 2 | | | | |
| $p = 4, k1 = 0.42, d = 1$ | | | | | $p = 4, k1 = 0.51, d = 1$ | | | | |
| $\left[\begin{array}{ccccc} 1.00 & -0.01 & 0.00 & 0.00 & 0.05 \\ 0.01 & 0.99 & 0.00 & 0.00 & -0.01 \\ -1.09 & 0.53 & 0.84 & -0.05 & 4.73 \\ -0.91 & 0.97 & -0.01 & 0.90 & 0.38 \\ -0.03 & 0.03 & 0.00 & 0.00 & 0.95 \end{array} \right]$ | | | | | $\left[\begin{array}{ccccc} 0.98 & 0.00 & 0.00 & 0.00 & -0.08 \\ 0.01 & 0.99 & 0.00 & 0.00 & -0.15 \\ -2.75 & 0.54 & 0.96 & -0.07 & -2.42 \\ -5.48 & 3.46 & 0.01 & 0.87 & -12.31 \\ -0.01 & 0.02 & 0.00 & 0.00 & 0.88 \end{array} \right]$ | | | | |
| Sub-period 3 | | | | | Sub-period 4 | | | | |
| $p = 5, k1 = 0.64, d = 2$ | | | | | $p = 4, k1 = 0.53, d = 1$ | | | | |
| $\left[\begin{array}{ccccc} 0.99 & 0.02 & 0.00 & 0.00 & 0.02 \\ -0.01 & 0.97 & 0.00 & 0.00 & -0.02 \\ 0.03 & -0.86 & 1.00 & -0.02 & -1.47 \\ 0.48 & -0.53 & 0.05 & 0.81 & -2.87 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.86 \end{array} \right]$ | | | | | $\left[\begin{array}{ccccc} 0.23 & -0.12 & 0.02 & 0.00 & -0.79 \\ -0.12 & 0.85 & 0.00 & 0.00 & 0.09 \\ -2.91 & 2.33 & 0.90 & 0.11 & -3.37 \\ -1.11 & -5.99 & 0.21 & 0.66 & 6.88 \\ 0.02 & 0.10 & 0.00 & 0.00 & 0.75 \end{array} \right]$ | | | | |

Table 6: FM-VAR causal structure. Non-causality from variable j in equation i . Main entries are the Wald statistics and the p-values (between parentheses)

| Sub-period 1 | | | | | | |
|---|-----------------|----------------|----------------|----------------|----------------|------------------|
| $p = 4, k1 = 0.42, d = 0, AIC = -37.41, BIC = -34.76$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div4r_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | $\Delta_{12}p_t$ |
| $divr_t$ | | ***** | 9.339 (0.053) | 2.667 (0.615) | 8.001 (0.091) | 4.243 (0.374) |
| $indpro_t$ | | 2.947 (0.566) | ***** | 5.742 (0.219) | 0.456 (0.977) | 1.840 (0.765) |
| $irst_t$ | | 15.712 (0.003) | 3.420 (0.490) | ***** | 1.425 (0.839) | 5.222 (0.265) |
| $irlt_t$ | | 3.851 (0.426) | 4.570 (0.334) | 2.595 (0.628) | ***** | 3.239 (0.519) |
| $\Delta_{12}p_t$ | | 3.643 (0.456) | 14.311 (0.006) | 8.770 (0.067) | 18.793 (0.000) | ***** |
| Sub-period 2 | | | | | | |
| $p = 4, k = 0.51, d = 0, AIC = -34.57, BIC = -30.59$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div4r_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | $\Delta_{12}p_t$ |
| $divr_t$ | | ***** | 10.150 (0.038) | 9.497 (0.049) | 5.347 (0.235) | 3.853 (0.426) |
| $indpro_t$ | | 9.143 (0.057) | ***** | 1.084 (0.896) | 1.857 (0.762) | 7.823 (0.098) |
| $irst_t$ | | 4.672 (0.322) | 0.935 (0.919) | ***** | 13.708 (0.008) | 1.201 (0.878) |
| $irlt_t$ | | 6.879 (0.142) | 7.296 (0.121) | 1.106 (0.893) | ***** | 3.534 (0.473) |
| $\Delta_{12}p_t$ | | 1.820 (0.768) | 1.358 (0.851) | 1.737 (0.783) | 0.682 (0.953) | ***** |
| Sub-period 3 | | | | | | |
| $p = 5, k1 = 0.64, d = 1, AIC = -39.98, BIC = -36.84$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div4r_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | $\Delta_{12}p_t$ |
| $divr_t$ | | ***** | 17.434 (0.004) | 12.893 (0.024) | 8.559 (0.128) | 2.642 (0.754) |
| $indpro_t$ | | 8.647 (0.124) | ***** | 9.444 (0.093) | 4.147 (0.528) | 3.362 (0.644) |
| $irst_t$ | | 1.803 (0.875) | 17.691 (0.003) | ***** | 13.947 (0.016) | 2.431 (0.787) |
| $irlt_t$ | | 3.103 (0.684) | 3.613 (0.606) | 17.441 (0.004) | ***** | 3.821 (0.575) |
| $\Delta_{12}p_t$ | | 2.419 (0.788) | 7.862 (0.164) | 15.322 (0.009) | 0.756 (0.981) | ***** |
| Sub-period 4 | | | | | | |
| $p = 4, k1 = 0.53, d = 0, AIC = -34.57, BIC = -30.59$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div4r_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | $\Delta_{12}p_t$ |
| $divr_t$ | | ***** | 33.852 (0.000) | 23.921 (0.000) | 7.430 (0.115) | 17.023 (0.002) |
| $indpro_t$ | | 0.810 (0.937) | ***** | 6.864 (0.143) | 4.300 (0.366) | 0.511 (0.972) |
| $irst_t$ | | 9.664 (0.046) | 32.042 (0.000) | ***** | 7.938 (0.094) | 1.932 (0.748) |
| $irlt_t$ | | 1.572 (0.813) | 9.187 (0.056) | 7.265 (0.122) | ***** | 5.035 (0.284) |
| $\Delta_{12}p_t$ | | 11.601 (0.021) | 6.851 (0.144) | 2.047 (0.727) | 4.748 (0.314) | ***** |

Table 7: Non-causality tests in surplus-lag VARX models. Money defined as DM4

| Sub-period 1 | | | | |
|--------------|--------------------|---------------------------|----------------------------------|----------------|
| Model | Dependent Variable | Forcing Variable (Causal) | Exogenous Control Variables | Wald |
| ARX(3,3) | $div4r_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 7.294 (0.063) |
| ARX(3,2) | $indpro_t$ | $div4r_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 0.128 (0.937) |
| Sub-period 2 | | | | |
| ARX(3,3) | $div4r_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 9.083 (0.028) |
| ARX(3,3) | $indpro_t$ | $div4r_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 9.198 (0.027) |
| Sub-period 3 | | | | |
| ARX(2,5) | $div4r_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 13.364 (0.020) |
| ARX(2,5) | $indpro_t$ | $div4r_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 7.078 (0.214) |
| Sub-period 4 | | | | |
| ARX(2,2) | $div4r_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 30.752 (0.000) |
| ARX(7,2) | $indpro_t$ | $div4r_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 0.835 (0.658) |

Table 8: Long-term matrices A for the five sub-periods. Money defined as $div4mr_t$

| $y_t^2 = (div4mr_t, indpro_t, irst_t, irlt_t, \Delta_{12}p_t)'$ | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|
| Sub-period 1 | | | | | Sub-period 2 | | | | |
| $p = 5, k1 = 0.53, d = 0$ | | | | | $p = 4, k1 = 0.56, d = 1$ | | | | |
| $\begin{bmatrix} 1.00 & -0.01 & 0.00 & 0.00 & 0.04 \\ -0.05 & 1.00 & 0.00 & 0.00 & 0.07 \\ -6.15 & 3.84 & 0.66 & -0.32 & 14.73 \\ -2.61 & 2.51 & -0.01 & 0.78 & 1.00 \\ -0.03 & 0.03 & 0.00 & 0.00 & 0.95 \end{bmatrix}$ | | | | | $\begin{bmatrix} 0.89 & 0.12 & 0.00 & 0.00 & 0.01 \\ 0.03 & 0.94 & 0.00 & 0.00 & 0.16 \\ -5.74 & 5.31 & 0.89 & -0.21 & 1.35 \\ -13.76 & 13.51 & -0.13 & 0.55 & -3.80 \\ 0.08 & -0.08 & 0.00 & 0.00 & 0.88 \end{bmatrix}$ | | | | |
| Sub-period 3 | | | | | Sub-period 4 | | | | |
| $p = 4, k1 = 0.66, d = 1$ | | | | | $p = 6, k1 = 0.38, d = 2$ | | | | |
| $\begin{bmatrix} 0.73 & -0.07 & 0.00 & 0.00 & 0.13 \\ -0.08 & -0.14 & 0.00 & 0.00 & 0.31 \\ -14.51 & -5.40 & 0.63 & 0.38 & -38.19 \\ -52.18 & -11.07 & -0.03 & 1.01 & -1.63 \\ -0.20 & 0.09 & 0.00 & 0.00 & -0.28 \end{bmatrix}$ | | | | | $\begin{bmatrix} 0.88 & -0.01 & 0.00 & 0.00 & -0.17 \\ 0.01 & 0.91 & 0.00 & 0.00 & -0.15 \\ -1.84 & -3.86 & 1.06 & -0.01 & -7.19 \\ -0.85 & 1.32 & 0.00 & 0.85 & 8.52 \\ 0.04 & 0.09 & 0.00 & 0.00 & 1.00 \end{bmatrix}$ | | | | |
| Sub-period 5 | | | | | | | | | |
| $p = 4, k1 = 0.49, d = 1$ | | | | | | | | | |
| $\begin{bmatrix} 0.99 & 0.02 & 0.00 & 0.00 & 0.02 \\ -0.01 & 0.97 & 0.00 & 0.00 & -0.02 \\ 0.03 & -0.86 & 1.00 & -0.02 & -1.47 \\ 0.48 & -0.53 & 0.05 & 0.81 & -2.87 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.86 \end{bmatrix}$ | | | | | | | | | |

Table 9: FM-VAR causal structure. Non-causality from variable j in equation i . Main entries are the Wald statistics and the p-values (between parentheses)

| Sub-period 1 | | | | | | |
|---|-----------------|----------------|----------------|----------------|----------------|------------------|
| $p = 5, k1 = 0.53, d = 0, AIC = -36.85, BIC = -33.46$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div4mr_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | Δ_{12p_t} |
| $div4mr_t$ | | ***** | 8.094 (0.151) | 8.413 (0.135) | 9.763 (0.082) | 3.085 (0.686) |
| $indpro_t$ | | 3.864 (0.569) | ***** | 5.722 (0.334) | 0.756 (0.979) | 2.832 (0.725) |
| $irst_t$ | | 18.791 (0.002) | 4.852 (0.434) | ***** | 6.055 (0.301) | 13.697 (0.0176) |
| $irlt_t$ | | 7.205 (0.205) | 14.268 (0.014) | 2.721 (0.742) | ***** | 6.636 (0.249) |
| Δ_{12p_t} | | 7.812 (0.166) | 10.993 (0.051) | 6.764 (0.238) | 19.267 (0.001) | ***** |
| Sub-period 2 | | | | | | |
| $p = 4, k = 0.56, d = 1, AIC = -36.93, BIC = -33.79$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div4mr_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | Δ_{12p_t} |
| $div4mr_t$ | | ***** | 32.972 (0.000) | 29.359 (0.000) | 23.865 (0.000) | 7.307 (0.120) |
| $indpro_t$ | | 4.628 (0.327) | ***** | 1.395 (0.845) | 0.880 (0.927) | 6.607 (0.158) |
| $irst_t$ | | 5.233 (0.264) | 3.645 (0.456) | ***** | 14.925 (0.005) | 1.230 (0.873) |
| $irlt_t$ | | 19.627 (0.000) | 20.532 (0.000) | 5.616 (0.229) | ***** | 0.802 (0.938) |
| Δ_{12p_t} | | 10.521 (0.032) | 8.931 (0.062) | 11.699 (0.019) | 9.021 (0.061) | ***** |
| Sub-period 3 | | | | | | |
| $p = 4, k1 = 0.66, d = 1, AIC = -39.29, BIC = -35.08$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div4mr_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | Δ_{12p_t} |
| $div4mr_t$ | | ***** | 2.759 (0.598) | 8.181 (0.085) | 6.682 (0.154) | 2.815 (0.589) |
| $indpro_t$ | | 1.843 (0.764) | ***** | 7.287 (0.121) | 9.642 (0.046) | 3.399 (0.493) |
| $irst_t$ | | 1.825 (0.767) | 3.991 (0.407) | ***** | 13.066 (0.011) | 1.075 (0.898) |
| $irlt_t$ | | 7.526 (0.111) | 8.094 (0.088) | 8.841 (0.065) | ***** | 14.400 (0.006) |
| Δ_{12p_t} | | 6.808 (0.146) | 2.569 (0.632) | 4.648 (0.325) | 12.899 (0.012) | ***** |
| Sub-period 4 | | | | | | |
| $p = 6, k1 = 0.38, d = 2, AIC = -39.10, BIC = -33.74$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div4mr_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | Δ_{12p_t} |
| $div4mr_t$ | | ***** | 14.287 (0.026) | 11.239 (0.081) | 10.350 (0.110) | 13.735 (0.032) |
| $indpro_t$ | | 7.529 (0.274) | ***** | 15.374 (0.017) | 6.088 (0.413) | 9.588 (0.143) |
| $irst_t$ | | 8.160 (0.226) | 6.767 (0.343) | ***** | 9.969 (0.125) | 6.345 (0.385) |
| $irlt_t$ | | 1.788 (0.938) | 7.920 (0.244) | 3.506 (0.743) | ***** | 6.332 (0.387) |
| Δ_{12p_t} | | 4.359 (0.628) | 11.571 (0.072) | 16.518 (0.011) | 4.166 (0.641) | ***** |
| Sub-period 5 | | | | | | |
| $p = 4, k1 = 0.49, d = 1, AIC = -32.61, BIC = -29.29$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div4mr_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | Δ_{12p_t} |
| $div4mr_t$ | | ***** | 10.887 (0.027) | 7.746 (0.101) | 8.240 (0.083) | 4.410 (0.353) |
| $indpro_t$ | | 1.816 (0.769) | ***** | 15.805 (0.003) | 2.134 (0.711) | 0.344 (0.986) |
| $irst_t$ | | 6.384 (0.172) | 41.646 (0.000) | ***** | 5.067 (0.280) | 2.339 (0.373) |
| $irlt_t$ | | 7.803 (0.099) | 4.577 (0.333) | 3.495 (0.478) | ***** | 8.925 (0.063) |
| Δ_{12p_t} | | 6.085 (0.192) | 9.969 (0.041) | 3.243 (0.518) | 4.143 (0.387) | ***** |

Table 10: Non-causality tests in surplus-lag VARX models. Money defined as DM4-

| Model | Dependent Variable | Forcing Variable (Causal) | Exogenous Control Variables | Wald |
|--------------|--------------------|---------------------------|----------------------------------|---------------|
| Sub-period 1 | | | | |
| ARX(2,2) | $div4mr_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 4.389 (0.111) |
| ARX(6,2) | $indpro_t$ | $div4mr_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 0.238 (0.887) |
| Sub-period 2 | | | | |
| ARX(2,2) | $div4mr_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 6.778 (0.033) |
| ARX(2,2) | $indpro_t$ | $div4mr_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 1.377 (0.502) |
| Sub-period 3 | | | | |
| ARX(2,2) | $div4mr_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 1.866 (0.393) |
| ARX(2,2) | $indpro_t$ | $div4mr_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 0.389 (0.823) |
| Sub-period 4 | | | | |
| ARX(2,3) | $div4mr_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 5.109 (0.163) |
| ARX(2,2) | $indpro_t$ | $div4mr_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 0.402 (0.817) |
| Sub-period 5 | | | | |
| ARX(2,2) | $div4mr_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 5.177 (0.075) |
| ARX(2,2) | $indpro_t$ | $div4mr_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 2.241 (0.326) |

Table 11: Long-term matrices A for the four sub-periods, money defined as $div3r_t$

| $y_t^3 = (div3r_t, indpro_t, irst_t, irlt_t, \Delta_{12}p_t)'$ | | | | | | | | | |
|---|--|--|--|--|---|--|--|--|--|
| Sub-period 1 $p = 5, k1 = 0.52, d = 0$ | | | | | Sub-period 2 $p = 4, k1 = 0.32, d = 1$ | | | | |
| $\begin{bmatrix} 1.01 & 0.00 & 0.00 & 0.00 & 0.05 \\ -0.05 & 1.00 & 0.00 & 0.00 & 0.07 \\ -5.71 & 3.49 & 0.68 & -0.32 & 14.87 \\ -2.16 & 2.19 & 0.00 & 0.79 & 0.91 \\ -0.03 & 0.03 & 0.00 & 0.00 & 0.95 \end{bmatrix}$ | | | | | $\begin{bmatrix} 0.96 & 0.04 & 0.00 & 0.00 & 0.00 \\ 0.04 & 0.94 & 0.00 & 0.00 & 0.12 \\ 1.28 & -1.81 & 1.01 & 0.01 & 1.86 \\ -1.14 & 1.13 & -0.02 & 0.97 & -7.11 \\ 0.02 & -0.02 & 0.00 & 0.00 & 0.99 \end{bmatrix}$ | | | | |
| Sub-period 3 $p = 5, k1 = 0.47, d = 0$ | | | | | Sub-period 4 $p = 4, k1 = 0.50, d = 1$ | | | | |
| $\begin{bmatrix} 0.98 & 0.01 & 0.00 & 0.00 & -0.05 \\ 0.00 & 0.99 & 0.00 & 0.00 & -0.05 \\ -0.11 & -0.32 & 0.99 & -0.06 & 0.46 \\ -0.62 & 0.43 & 0.01 & 0.84 & 3.67 \\ 0.00 & -0.01 & 0.00 & 0.00 & 0.88 \end{bmatrix}$ | | | | | $\begin{bmatrix} 0.70 & 0.13 & -0.01 & 0.00 & -0.26 \\ -0.15 & 0.49 & 0.02 & 0.00 & 0.14 \\ 4.76 & 7.98 & 0.40 & 0.08 & 1.27 \\ -1.59 & -4.18 & 0.00 & 0.53 & 6.07 \\ 0.05 & 0.04 & 0.00 & 0.00 & 0.90 \end{bmatrix}$ | | | | |

Table 12: FM-VAR causal structure. Non-causality from variable j in equation i . Main entries are the Wald statistics and the p-values (between parentheses)

| Sub-period 1 | | | | | | |
|---|-----------------|----------------|----------------|----------------|----------------|------------------|
| $p = 5, k1 = 0.52, d = 0, AIC = -36.96, BIC = -33.57$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div3r_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | $\Delta_{12}p_t$ |
| $div3r_t$ | | ***** | 7.392 (0.193) | 9.786 (0.081) | 9.392 (0.094) | 4.195 (0.521) |
| $indpro_t$ | | 4.545 (0.474) | ***** | 5.431 (0.365) | 0.741 (0.980) | 3.287 (0.655) |
| $irst_t$ | | 18.738 (0.002) | 4.364 (0.498) | ***** | 5.635 (0.343) | 14.149 (0.015) |
| $irlt_t$ | | 6.181 (0.289) | 12.785 (0.025) | 2.682 (0.748) | ***** | 5.942 (0.311) |
| $\Delta_{12}p_t$ | | 7.332 (0.197) | 9.971 (0.079) | 6.956 (0.223) | 19.002 (0.002) | ***** |
| Sub-period 2 | | | | | | |
| $p = 4, k = 0.32, d = 1, AIC = -36.70, BIC = -33.94$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div3r_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | $\Delta_{12}p_t$ |
| $div3r_t$ | | ***** | 10.482 (0.033) | 10.849 (0.028) | 7.200 (0.125) | 4.141 (0.387) |
| $indpro_t$ | | 2.582 (0.629) | ***** | 0.939 (0.918) | 1.835 (0.765) | 5.434 (0.245) |
| $irst_t$ | | 1.189 (0.879) | 3.187 (0.526) | ***** | 10.033 (0.039) | 1.145 (0.887) |
| $irlt_t$ | | 3.957 (0.411) | 7.707 (0.103) | 2.645 (0.619) | ***** | 1.957 (0.743) |
| $\Delta_{12}p_t$ | | 1.495 (0.827) | 1.613 (0.806) | 1.391 (0.845) | 0.687 (0.952) | ***** |
| Sub-period 3 | | | | | | |
| $p = 5, k1 = 0.47, d = 0, AIC = -39.57, BIC = -34.85$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div3r_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | $\Delta_{12}p_t$ |
| $div3r_t$ | | ***** | 13.206 (0.021) | 16.187 (0.006) | 17.398 (0.004) | 10.587 (0.060) |
| $indpro_t$ | | 3.532 (0.618) | ***** | 5.576 (0.339) | 11.355 (0.044) | 6.375 (0.271) |
| $irst_t$ | | 8.370 (0.137) | 5.236 (0.387) | ***** | 18.991 (0.002) | 1.482 (0.915) |
| $irlt_t$ | | 1.911 (0.861) | 7.085 (0.214) | 5.199 (0.392) | ***** | 4.701 (0.453) |
| $\Delta_{12}p_t$ | | 3.301 (0.653) | 6.747 (0.240) | 9.971 (0.076) | 3.183 (0.672) | ***** |
| Sub-period 4 | | | | | | |
| $p = 4, k1 = 0.50, d = 0, AIC = -34.31, BIC = -30.03$ | | | | | | |
| $\downarrow i$ | $\rightarrow j$ | $div3r_t$ | $indpro_t$ | $irst_t$ | $irlt_t$ | $\Delta_{12}p_t$ |
| $div3r_t$ | | ***** | 10.769 (0.029) | 7.925 (0.094) | 6.692 (0.153) | 3.025 (0.553) |
| $indpro_t$ | | 1.630 (0.803) | ***** | 16.402 (0.002) | 3.272 (0.513) | 0.608 (0.962) |
| $irst_t$ | | 5.046 (0.282) | 32.016 (0.000) | ***** | 3.245 (0.517) | 2.807 (0.590) |
| $irlt_t$ | | 7.441 (0.114) | 3.822 (0.430) | 3.686 (0.450) | ***** | 8.737 (0.068) |
| $\Delta_{12}p_t$ | | 5.877 (0.208) | 8.292 (0.081) | 4.963 (0.291) | 4.369 (0.358) | ***** |

Table 13: Non-causality tests in surplus-lag VARX models. Money defined as DM3

| Sub-period 1 | | | | |
|--------------|--------------------|---------------------------|----------------------------------|----------------|
| Model | Dependent Variable | Forcing Variable (Causal) | Exogenous Control Variables | Wald |
| ARX(2,2) | $div3r_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 3.998 (0.135) |
| ARX(7,2) | $indpro_t$ | $div3r_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 0.486 (0.783) |
| Sub-period 2 | | | | |
| ARX(3,2) | $div3r_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 6.175 (0.045) |
| ARX(2,2) | $indpro_t$ | $div3r_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 1.653 (0.437) |
| Sub-period 3 | | | | |
| ARX(2,4) | $div3r_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 9.691 (0.045) |
| ARX(2,2) | $indpro_t$ | $div3r_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 0.781 (0.676) |
| Sub-period 4 | | | | |
| ARX(2,2) | $div3r_t$ | $indpro_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 14.988 (0.000) |
| ARX(2,2) | $indpro_t$ | $div3r_t$ | $irst_t, irlt_t, \Delta_{12}p_t$ | 1.260 (0.532) |